USN | | | | | | | | | | MATDIP401

Fourth Semester B.E. Degree Examination, December 2010

Advanced Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. Find the ratio in which the line joining (2, 4, 16) and (3, 5, −4) is divided by the plane $2x-3y+z+6=0$. **(06 Marks)**

b. Find the angle between the lines whose direction cosines are given by $3l+3+5n=0$ and $6mn-2ln+5lm=0$. **(07 Marks)**

c. Derive the equation of the plane in the form $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. **(07 Marks)**

2 a. Find the reflection of the point (1, 3, 4) in the plane $2x-y+z+3=0$. **(07 Marks)**

b. Find the equation of the line through (1, 2, -1) and perpendicular to each of the lines $\frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$ and $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$. **(06 Marks)**

c. Prove that the lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{1+z}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ intersect and find the coordinates of their point of intersection. **(07 Marks)**

3 a. If $\vec{A}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{B}=3\hat{i}-\hat{j}+2\hat{k}$ then :

i) Show that $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ are orthogonal and

ii) Find the angle between $2\vec{A}+\vec{B}$ and $\vec{A}+2\vec{B}$. **(07 Marks)**

b. Prove that $[\vec{A}+\vec{B},\vec{B}+\vec{C},\vec{C}+\vec{A}]=2[\vec{A}\vec{B}\vec{C}]$. **(06 Marks)**

c. If $\vec{A}=2\hat{i}-\hat{j}+3\hat{k}$, $\vec{B}=-\hat{i}+3\hat{j}+3\hat{k}$ and $\vec{C}=\hat{i}+\hat{j}-2\hat{k}$, find the reciprocal triad $(\vec{A'}\vec{B'}\vec{C'})$. **(07 Marks)**

4 a. For the curve $\vec{R}=a(\cos t\,\hat{i}+\sin t\,\hat{j}+t\tan\alpha\,\hat{k})$ where a and α are constants, evaluate $\left|\frac{d\vec{R}}{dt}\times\frac{d^2\vec{R}}{dt^2}\right|$. **(06 Marks)**

b. The position vector of a moving particle at time t is $\vec{R}=t^2\,\hat{i}-t^3\,\hat{j}+t^4\,\hat{k})$. Find the tangential and normal components of its acceleration at t = 1. **(07 Marks)**

c. Find the directional derivative φ = xyz along the direction of the normal to the surface $x^2z+y^2x+z^2y=3$ at the point (1, 1, 1). **(07 Marks)**

5 a. Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $r^2 = x^2 + y^2 + z^2$. **(07 Marks)**

b. If $\vec{F} = e^{xyz}(\hat{i} + \hat{j} + \hat{k})$ find $\text{div}\,\vec{F}$ and $\text{curl}\,\vec{F}$. **(06 Marks)**

c. Prove that $\nabla \times \nabla \times \vec{F} = \nabla(\nabla.\vec{F}) - \nabla^2 \vec{F}$. **(07 Marks)**

6 a. Prove that $L\{\cos at\} = \dfrac{s}{s^2 + a^2}$ $\quad s > 0$ **(05 Marks)**

b. Find : i) $L\{e^{-t}\sin^2 t\}$ ii) $L\{te^{-t}\sin 3t\}$ iii) $L\left\{\dfrac{\cos 2t - \cos 3t}{t}\right\}$ **(15 Marks)**

7 a. IF $f(t) = \begin{cases} t^2, & 0 < t < 2 \\ t-1, & 2 < t < 3 \\ 7, & t > 3 \end{cases}$, find $L\{f(t)\}$. **(07 Marks)**

b. Find $L^{-1}\left\{\dfrac{4s+5}{(s-1)^2(s+2)}\right\}$. **(06 Marks)**

c. Apply convolution theorem to evaluate $L^{-1}\left\{\dfrac{s}{(s^2+a^2)^2}\right\}$. **(07 Marks)**

8 a. If $f'(t)$ is a continuous function and $L\{f(t)\}=F(s)$ then prove that $L\{f'(t)\} = sF(s) - f(0)$. **(04 Marks)**

b. Solve the following using Laplace transform :
$y'' + 2y' - 3y = \sin t$, when $y(0) = 0 = y'(0)$. **(06 Marks)**

c. Using Laplace transform method, solve the simultaneous equations:
$\dfrac{dx}{dt} + 5x - 2y = t$; $\dfrac{dy}{dt} + 2x + y = 0$, given $x = y = 0$, when $t = 0$. **(10 Marks)**

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